MATH 504 HOMEWORK 3

Due Wednesday, October 3.

Problem 1. *Suppose that κ is strongly inaccessible. Show that:*

(1) *α is an ordinal iff $V_\kappa \models$ "α is an ordinal".*
(2) *α is a cardinal iff $V_\kappa \models$ "α is a cardinal".*
(3) *α is a regular cardinal iff $V_\kappa \models$ "α is a regular cardinal".*
(4) *α is strongly inaccessible iff $V_\kappa \models$ "α is strongly inaccessible".*

Note: the above problem shows that if κ is the least inaccessible cardinal, then $V_\kappa \models$"there are no inaccessible cardinals". It follows that it cannot be proved in ZFC that inaccessible cardinals exist.

Problem 2. *Suppose that κ is inaccessible. Show that $|V_\kappa| = \kappa$ and V_κ satisfies the Replacement axiom, i.e. show that if f is a function from a set $X \in V_\kappa$ into V_κ, then $f \in V_\kappa$.*

Problem 3. *Assume that $V = L$. Prove that $V_\alpha = L_\alpha$ iff $\alpha = \aleph_\alpha$. Here you can use the theorem that in $V = L$, GCH holds.*

Problem 4. *Suppose that κ is a regular uncountable cardinal in L. Show that L_κ satisfies the axioms of $ZF \setminus$ Powerset with the exception of Comprehension. (L_κ also satisfies Comprehension, but I will show that in class)*

Problem 5. *Suppose that $M \prec L_{\omega_1}$. Show that M is transitive. (Hint: for $X \in M$, take the $\prec_L$-least onto $f : \omega \to X$. Show that f is definable in L_{ω_1} from X and use this to show that $f \in M$. Also show $\omega \subset M$. Use these to prove that range of f is a subset of M)*